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METAMODELLING FE MODELS OF ANCHOR PILES DRIVEN IN SAND

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ABSTRACT

The paper presents a novel procedure to support the preliminary design of anchor piles for floating offshore wind facilities. As Floating Offshore Wind Turbines (FOWTs) operate in a complex marine environment, under extreme events, they would be subject to load actions from impetuous wind, steep waves and strong currents. Under these conditions, tensile loads transmitted by the FOWT structure may become critical for the piles, particularly when they are used with vertical moorings. The tensile capacity of piles driven in sand is commonly estimated with the use of cone penetration test methods (CPT-methods), while an insight into the deformation can be achieved with the use of load-transfer approaches. However, there are still uncertainties on what should be the most suitable formulation to be used among the available ones. The approach proposed in this work combines Finite Element (FE) and metamodeling techniques to analyse the response of pile anchors driven in sand and subjected to pull-out load. The FE model used in the study is a simple but effective solution to reproduce the response of anchor pile subject to monotonic load and is used to train a metamodel. Differently from FE model, which computational expense limits the use for probabilistic analyses, a metamodel can be used to perform some sensitivity analysis at negligible computational cost. The procedure on how to build a metamodel based on a reliable FE model is here illustrated for open-ended driven piles installed in sand and subjected to drained monotonic tensile loading.

Keywords: Anchor driven piles, FE model, metamodel, sand

1. INTRODUCTION

The paper proposes the application of metamodeling technique as a supporting tool for the preliminary design of offshore foundations for floating offshore wind turbines. Contrarily to the traditional bottom-fixed solutions used in shallow waters, floating structures are used in deep water sites. The floater is secured to the seabed through mooring lines and anchors for what significant experience derives from the O&G sector. However, the conditions for wind facilities are different and investment savings may arise from optimized anchors (Myhr et al., 2014).

In this work the case of driven pile anchors used for tension-leg platform floating structures is analysed. The piles are considered to be installed in homogeneous sand deposit, while the load deriving from the floater is a pure vertical tensile load. Traditional methods to estimate the pile capacity in offshore conditions are the well-known CPT-methods and a unified solution has been recently proposed (Lehane et al., 2020a). However, the information provided by these methods is limited to the pile capacity only. The pile in-service performance can be better investigated with the use of load-transfer methods for what several formulations are available (Bohn et al., 2017), but uncertainties arise when selecting the most suitable for a certain site condition. Finite Element (FE) models are an alternative and powerful solution to investigate the problem providing a large number of output quantities, which reliability is a function of the model complexity (Said et al., 2008; Yang et al.,2019; Han et al., 2020).

However, the use of these methods for practical design requires good experience by the designer. This might limit their usage, particularly when facing preliminary design activities where several variables

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must be considered and the designer wants a quick and easy-to-use solution. In this stage of the design, it is common to use probabilistic approaches to identify the most promising solution, and the FE model cannot be suitably coupled with these methods because of their computational cost.

The response of the computationally expensive model can be similarly reproduced by a new but fast-to-evaluate mathematical model named metamodel. The metamodel, or surrogate model, is a model of another model and it consists of an analytical relation between the model input and its output variables. The metamodel is trained by some input-output combinations of the numerical model. Once opportunely calibrated it can be used to reproduce the accurate numerical model response with an irrelevant computational cost, and its application in the civil engineering context has been proved (Toe et al., 2018; Marelli and Sudret, 2018; Lambert et al., 2021). Various types of metamodels can be used to the scope, from complex Neural Network (Papadrakis et al., 1998) and Kriging (Martin and Simpson, 2005) methods, to easier-to-construct polynomial models (Fang and Horstemeyer, 2006), rather than Support Vector Regression models (Clarke et al., 2005). In this work, the Polynomial Chaos Expansion (PCE) method is adopted (Sudret, 2008).

In this paper, the use of the PCE metamodel is tested in the offshore geotechnical field. A numerical strategy to build a 2D FE model of the anchor pile installed in a homogeneous sand deposit is developed. The strategy allows to define the metamodel, with the identification of opportune input parameters and output indicators to be estimated. A parametric study is carried out defining a sort of ‘numerical experimental’ database. The input-output combinations of this study are used to generate the PCE metamodel. The PCE accuracy is measured and its reliability as a predictive tool for the pile design is tested by comparing it with some experimental data from the literature.

The PCE metamodel is tested for a simple FE model but proved able to replicate its response with good accuracy and might be extended to more complex case studies following the same approach.

2. THE POLYNOMIAL CHAOS EXPANSION (PCE) METAMODEL TECHNIQUE

In engineering design, the physical systems are commonly represented by a mathematical model which depends on the combination of different input parameters. The model will provide a certain response through the use of simple analytical formulas or complex sets of partial differential equations. The latter are generally solved by using specific numerical methods, like the finite difference or finite element methods. However, the input variables that characterise the model (i.e., geometry, material properties, loads, etc.) can have many uncertainties, which in turns result in a relevant randomness of the model response. This is particularly true for any offshore foundation design problem. The numerical model can then be used to investigate the system response to various conditions, but its use remains limited in the design context because of the time cost of the simulations.

In the following the main characteristics of the PCE are illustrated and the method for calibrating and validating the metamodels are introduced.

2.1 Details of the PCE

Given a certain physical system, its behaviour may be ideally represented by a mathematical model *f*, that provides a deterministic relation between the system’s input and output. Now, denoted $\boldsymbol{x} \equiv \{x_1, \dots, x_N\}^T$ the set of input parameters of the problem, the evaluation $f(\boldsymbol{x})$ would return the set of output quantities $\boldsymbol{y} \equiv \{y_1, \dots, y_M\}^T$. When considering a FE model, $\boldsymbol{x}$ may be related to its geometry rather than to the material properties or any other model parameter. On the other hand, the output $\boldsymbol{y}$, generally named model response vector, can be a collection of some model behavioural features (e.g., reaction forces, strains and stresses at relevant nodes of the mesh, etc.). The model *f* cannot have an explicit expression, but it can be approximated by a new function, g, that is the polynomial chaos expansion of the model response vector (Blatman, 2009). The polynomial chaos decomposition of the original model is defined as the linear combination of selected multivariate orthonormal basis $\left(\psi_k(\boldsymbol{x})\right)_{k\in\mathbb{N}}$ and their corresponding coefficients α_k as:

$$g(\boldsymbol{x}) = \sum_{k\in K} \alpha_k \psi_k(\boldsymbol{x}) \approx y \tag{1}$$

where $K \in \mathbb{N}$ is the number of terms in the expansion and the PCE function is here illustrated for a generic output variable (i.e., $M = 1$). The polynomial basis obeys an orthogonal and normal rule, that is their inner product is $\langle\psi_n, \psi_m\rangle = \delta_{nm}$, where δ_{nm} is the Kronecker delta function. Each family of orthonormal polynomials is associated to a certain distribution type (e.g., Hermite polynomials for Standard Gaussian distribution) and should be selected according to the input measure. Once an appropriate set of basis functions is selected, the problem reduces to the determination of the coefficients α_k. They have to be computed in order to minimize the distance between the model function f(x) and its approximation g(x). A classical least-square analysis can be carried out to compute the coefficients:

$$\boldsymbol{\alpha} = (\alpha_k)_{k \in K} = \underset{\alpha \in \mathbb{R}^K}{\operatorname{argmin}} \, \mathbb{E}\left[\left(f(x) - \sum_{k \in K} \alpha_k \psi_k(x)\right)^2\right] \tag{2}$$

where $\mathbb{E}$ is the mean operator.

2.2 Training the PCE

In order to train or calibrate the PCE of a FE model a certain number of input-output combinations is required to create the input vector $\boldsymbol{x}$ and the model evaluations $f(\boldsymbol{x})$ given by the FE simulations. Once identified the problem position, that means the selection of the problem's input and the relevant output to be analysed, a strategy should be adopted for sampling the input variables within reasonable ranges. Then, defined n the size of the training sample, a PCE_n can be built by using Eq. (1) and Eq. (2) and its accuracy will be a function of n.

In the following the Latin Hypercube Sampling technique (LHS, McKay et al., 1979) is used to create the sample. It is a space-filling sampling method for what each variable is divided within its range of variability into a number of intervals equal to the sample size. Then, the sample points are randomly selected within the interval in order to have a uniform distribution of each input variable in their domains. Contrarily to the popular Monte Carlo Simulation method, that is a random sampling method, the LHS allows to have a randomly defined sample while assuring an optimum and uniform coverage of the input variables. This choice would lead to an expected uniform accuracy of the trained PCE within the whole domain of each input parameter.

2.3 The Leave-One-Out Cross-Validation method

After defining the method to build the metamodel a validation strategy to assess its reliability must also be defined before allowing its use as a surrogate of the FE model. An efficient strategy to validate any metamodel is the Cross-Validation (CV) method and in this work the Leave-One-Out (LOO) option is adopted.

The CV technique (Stone, 1974) consists on dividing the defined sample into two subsamples. A metamodel is then built from one subsample, which can be named as the training set, and its performance is evaluated by comparing with the other sample, which in turn is the validation set. In the LOO method, the training set contains all but one of the input-output combinations of the original sample. The latter being used as the validation set to compute the metamodel accuracy.

Named $\boldsymbol{x}^{(i)}$ the validation set, the metamodel is built for the remaining training set and the predicted residual of the i-th observation is defined as:

$$\Delta^{(\boldsymbol{i})} = f\left(\boldsymbol{x}^{(i)}\right) - g^{(-i)}\left(\boldsymbol{x}^{(i)}\right) \tag{3}$$

Which is basically the difference between the FE observation and the PCE prediction in our case. The expected error of the metamodel is then computed for the whole sample size and the LOO error is defined as:

$$Err_{LOO} = \frac{1}{N}\sum_{1}^{N}\left(\Delta^{(i)}\right)^2 \tag{4}$$

However, in practice it is common to compute the normalized LOO error defined as:

$$\varepsilon_{LOO} = \frac{Err_{LOO}}{\mathrm{COV}\big(f(x)\big)} \tag{5}$$

Where the empirical covariance of the output quantity is considered. The normalized LOO error is also commonly reported as $Q^2 = 1 - \varepsilon_{LOO}$, that is the predictive capacity factor of the metamodel.

3. THE FINITE ELEMENT TESTING PROGRAMME

The paper focuses on the monotonic response of steel open-ended anchor driven piles installed in homogeneous sand deposit and subjected to a pure vertical tensile load. A numerical strategy is first defined to build the FE model of the pile by using six input parameters. Then, according to the LHS technique, a sample of these input variables is created and the simulations are run. The results of the parametric testing program are illustrated and the output variables for training the metamodels are identified.

3.1 FE model details

The pile FE model is represented with a 2-dimensional axial-symmetry, thanks to the geometrical and loading conditions of the considered case study. The model is built considering a wished-in-place pile where the installation effects deriving from pile driving are implemented in the soil stress state. The pile is installed in sand and total stress, small-strain, static simulations are performed. In this context, the problem position is defined to build the FE model (i.e., the *f* function) starting from six input variables that are resumed in Table 1.

Table 1. Input parameters of the FE model and range of variation for the testing programme.

Input parameter	Symbol	Unit	Range	Chow, 1997	Rücker et al. 2013
Pile diameter	D	m	0.25 – 1.00	0.324	0.711
Slenderness ratio	L/D	-	10 – 60	34.88	24.77
Thickness ratio	D/t	-	15 – 80	25.51	56.88
Soil density	D_r	%	10 – 100	66.62	57.00
Water level	λ_w	-	0.0 – 1.0	0.275	0.02
Interface angle	δ	degree	20.0 – 35.0	26.8	29.0

The first three concerns the pile geometry and both length and pile wall thickness are normalized by the diameter. Then, there are two parameters of the site conditions. As said, a homogeneous deposit is considered in the study and a uniform soil relative density is selected as problem input. The water level parameter is introduced to account for the location of the water table level in the site, and its depth with respect to the ground surface is defined as function of the model height (i.e., $z_w = \lambda_w*(L+10D)$). This parameter is considered because both the FE and PCE models are then tested with some data from literature that are not in offshore sites.

The FE model is built starting from the geometrical input that define the model geometry, which extends laterally for 15 diameters and vertically for 10 diameters below the pile base. Horizontal and vertical restraints are then applied at the complementary outside sides. The pile is modelled as a uniform solid assuming a fully plug failure and a linear elastic material is assigned with equivalent properties computed according to its cross section. A uniform mesh having size of D/8 is used in the area close to the pile perimeter, while it then becomes coarser moving to the external model bounds as illustrated in Figure 1.

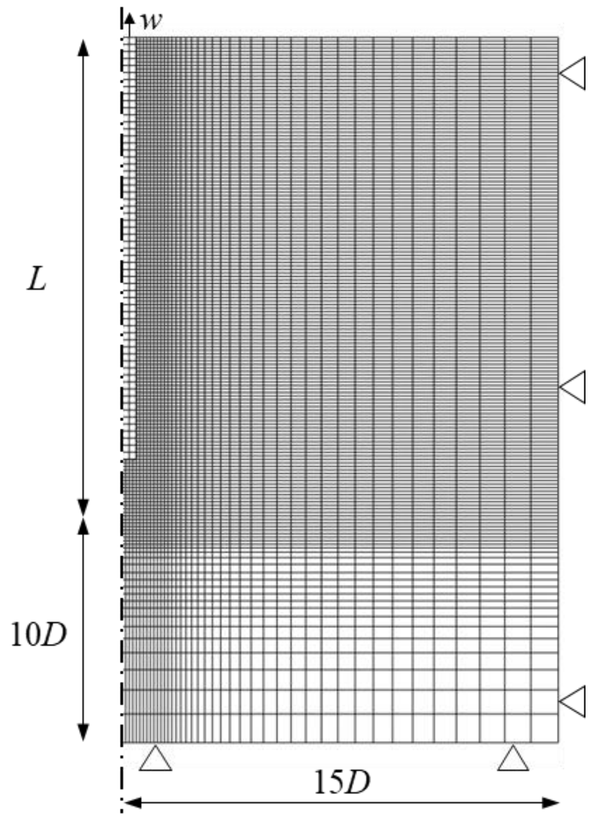

Figure 1. FE model of the anchor pile and boundary conditions.

The sand deposit is applied a linear elastic constitutive model and obeys a Drucker-Prager plastic failure criterion. The soil properties are assigned as function of an artificial cone resistance that is derived from the two site input parameters. The cone resistance is computed by considering the assumption of a homogeneous deposit (i.e., D_r = constant) according to the equation from Jamiolkowski et al. (2003):

$$q_{c,FE} = p_a \cdot 17.61 \cdot \left(\frac{p'_0}{p_a}\right)^{0.50} \cdot exp(3.10 \cdot D_r) \tag{6}$$

Where p_a=101.3kPa and p'_0 is computed considering an earth pressure coefficient at rest $K_o = 1 - sin\phi_{cv}$ and $\phi'_{cv} = 32^o$. The artificial cone resistance allows to assign all the soil properties to vary with depth in the model, thus accounting for the stress state variation. From the cone resistance data, the elastic small-strain shear modulus of the soil is computed using the equation modified by Chow (1997) as:

$$G_{max} = q_c \cdot [A + B\eta - C\eta^2]^{-1} \tag{7}$$

Where A = 0.0203; B = 0.00125; C = 1.216x10^{-6}; $\eta = q_c(p_a\sigma'_v)^{-0.5}$ and the soil Poisson's ratio is considered equal to 0.2. The soil plastic strength parameters are then computed using the well-known relationship proposed by Kulhawy and Mayne (1990) for the soil peak friction angle and computing the dilatancy according to the critical state angle.

As for the soil-pile interface property, a simple Coulomb-like frictional law is assigned with a penalty coefficient given by the tangent of the last input parameter provided in Table 1. The final step for building the pile FE model consists of assigning a soil stress state that would consider the effect due to the pile driving installation. Therefore, the radial stress around the pile shaft is modified according to the recently proposed Unified CPT-method (Lehane et al., 2020a).

3.2 Results of the FE testing program

The FE model of any driven pile installed in a homogeneous sand deposit can be built by following the simple procedure described above and using any combination of the six input parameters identified. The procedure is used to produce the input/output data required to calibrate the metamodel. First, a sample is built using the LHS technique. Different ranges for each input are identified as described in Table 1. They are selected to cover the geometrical properties of existing pile database, which are then used to test the PCE prediction capacity. A minimum value of 10% is set for the soil relative density, while water table is free to vary from dry to fully saturated condition. The interface angle is varied within the range defined by Han et al. (2018) after studying the interface friction strength as function of different sand-steel properties. A sample of size 100 is created for the input, the FE models are built and relevant simulations run. The results are illustrated in Figure 2 with curves reporting the normalised vertical load ($\bar{V} = V/(D \int \sigma'_v dz)$) and displacement ($\bar{w} = w/D$).

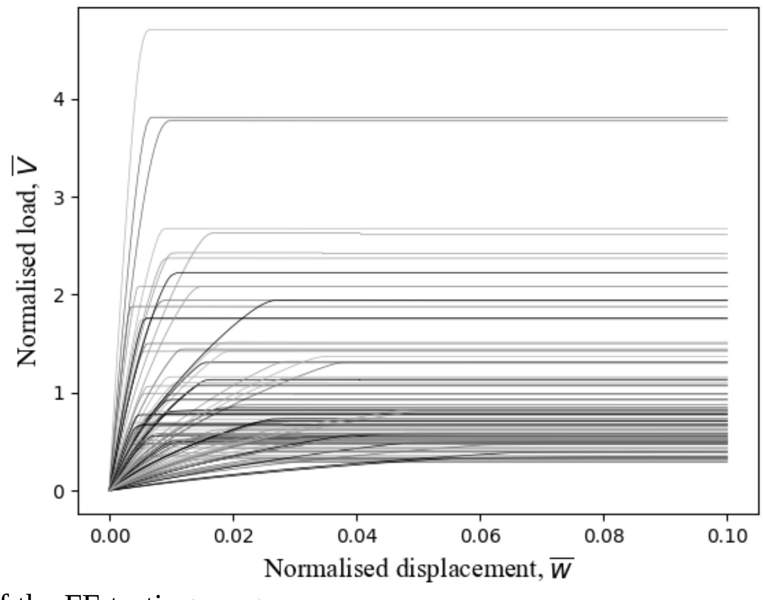

Figure 2. Results of the FE testing program.

The FE results are the analysed to collect the output quantities that form the model response vector, y. According to the load-displacement results, two outputs are identified: the ultimate load and the curve stiffness at a load rate of 50% the ultimate capacity that was recognized as a service level by Lehane et al. (2020b). The two outputs are thus collected by each simulation and gathered in the model response vector to calibrate the PCE metamodel.

4. PCE ASSESSMENT AND EXPLOITATION

4.1 Accuracy of the PCE

Given the 100 data of the LHS sample, two PCE metamodels are built by using Eq. (2) to compute their coefficients. The first is trained to evaluate the ultimate pull-out capacity of the driven pile and the second to estimate the load-displacement stiffness at the defined service load level. The input data generated by the LHS method have a uniform distribution and the Legendre polynomial basis are used in the computations. The metamodels are built with the LOO method, therefore the PCE are generated by using training set containing 99 data. Their evaluations, compared to the FE simulation results, are reported in Figure 3 where the best fit line is represented by a continuous black line.

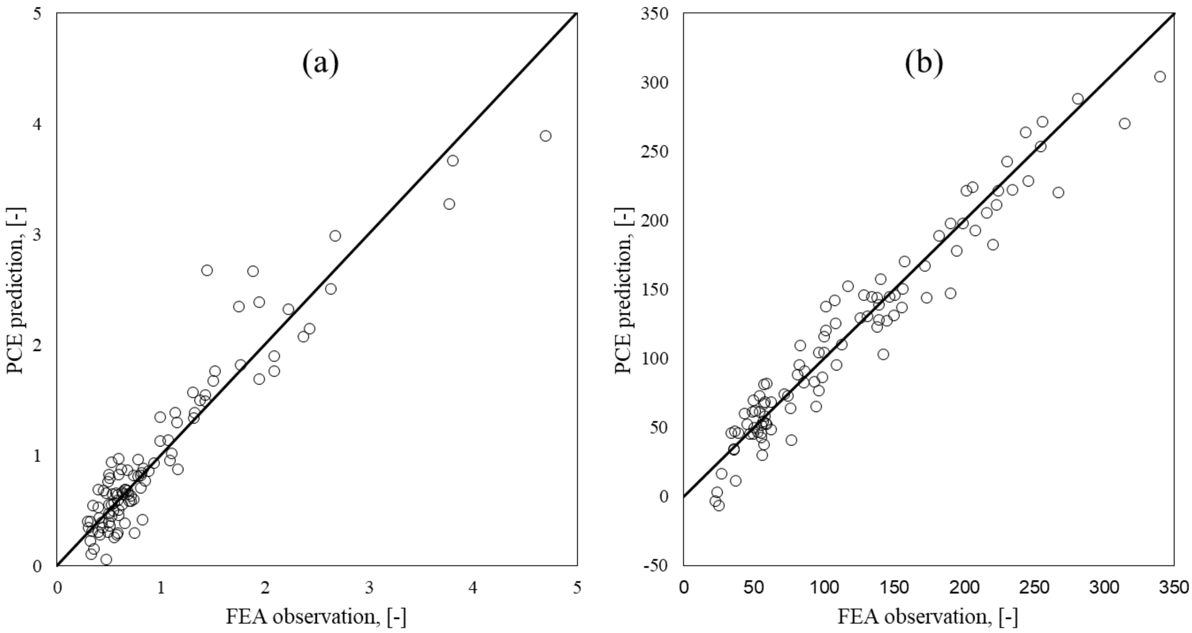

Figure 3. PCE predictions vs FE observations for: (a) normalized ultimate pile capacity, $\bar{V}$, and (b) normalized stiffness, $\bar{K} = \bar{V}/\bar{w}$, at service load level (50% of $\bar{V}$).

The two plots only provide a qualitative representation of the metamodels prediction capacities, but their effectiveness is measured through the LOO errors defined in Section 2. A normalized error of 0.110 (Err_{LOO} = 0.069) is measured for the limit capacity PCE, while it reduces to 0.060 (Err_{LOO} = 327.46) for the stiffness outcome. The use of larger sample size for training the two PCE metamodels might lead to improved capacity of predictions, but the obtained measures are considered sufficient in this study.

4.2 PCE vs FE predictions of experimental evidence

Once assessed the PCE metamodels can be used, for any input combination, as predictive tools of the two output indicators. Here a rapid demonstration is proposed by considering two experimental data available in the literature (Chow, 1997; Rücker et al. 2013) and for what the measured input data are reported in Table 1. All the input parameters of the two considered piles are given in the corresponding papers. Being natural deposits, they are not perfectly homogeneous as considered in the FE approach of this work. Therefore, the average soil relative densities reported in Table 1 for each site have been estimated from the cone tip profiles given in the references and using Eq. (6).

The two PCE are used to evaluate the outputs by entering the input data. Similarly, the modelling procedure described in section 3.1 is applied to build the FE models of the two piles. The results of the two models (i.e., the FE model and its surrogate PCE) are reported in Figure 4 together with the experimental data. The PCE can produce a simple bi-linear curve only given the two selected output that is able to estimate, but its shape is very similar to the curve predicted by the simplified FE model.

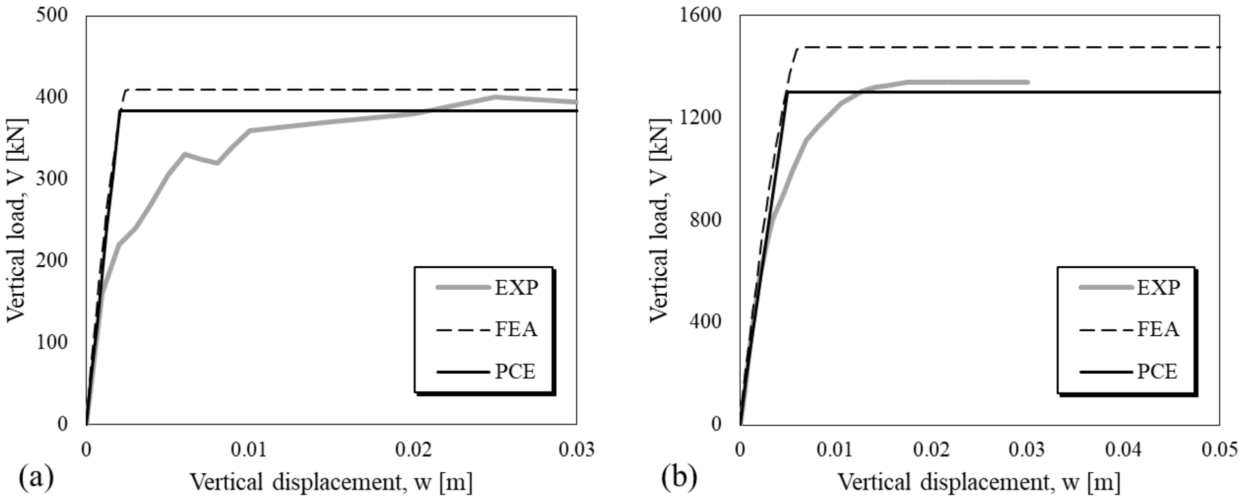

Figure 4. Comparison of PCE and FE evaluations of the experimental load-displacement curves of: (a) Chow (1997) and (b) Rücker et al., (2013).

In terms of prediction accuracy, the PCE shows very good comparison with the FE approach that has trained the metamodel. The limit capacity is slightly underestimated in both cases, with relative errors of 6.22% and 11.70% for case (a) and (b), respectively. The measured errors are in accordance with the general normalized LOO error of prediction of the PCE. Similarly, the stiffness outcome is evaluated with an error of 11.22% for case (b) that reduces to only 1.22% in case (a). These estimations are again in accordance with the normalized error computed with the LOO approach.

Finally, it is worth noticing that the comparison with the experimental data is excellent. The PCE metamodel proved able to predict the experimental response of the two piles with a very small error. At the same time, it has to be remembered that the PCE accuracy is function of the FE model reliability. In fact, the results of Figure 4 illustrate how the FE approach adopted for the study can also provide reliable estimation of the experimental data, thus similar accuracy is expected by its surrogate model.

5. CONCLUSIONS

A method for supporting the design of offshore foundation system has been proposed. The approach is alternative to the CPT and load-transfer methods commonly used for the design, as it makes use of the metamodeling (often referred as “surrogate” models) technique coupled to FE modelling.

The research has particularly focused on the design of anchoring systems for tension-leg platforms, with the scope of providing a novel procedure for the preliminary design of the foundations of floating offshore wind turbines. The main target of the procedure is to provide a robust, time and cost-effective tool. Above all the possible anchoring solutions, the response of driven piles in sand is analysed but the procedure could be similarly extended to other anchor types.

The PCE metamodel type is used to estimate some behavioural features of the FE model. It creates an approximation function that represents the correlation between the input parameters of the FE model and its output. The approximation function then provides a deterministic algorithm for any input combination and can be easily coupled to traditional probabilistic approaches that are commonly employed in preliminary design.

The metamodel is built for a simple but effective FE model of the pile anchors and proves to lead to accurate results with only 100 input data used for its calibration. In fact, it is tested as predictive tool to replicate some experimental tests available in literature.

The obtained results have highlighted that:

1. The approach is promising, and a good prediction capacity is obtained for a very small amount of data.
2. The PCE accuracy can be improved by simply increasing the training sample size or by varying some feature of the metamodel (i.e., input distribution and polynomial basis; integration strategy to compute coefficients; etc.).
3. The outcome of the approach is a mathematical equation that can be used for any input combination.

The approach provides an alternative to the use of complex and time-cost expensive FE models. The metamodel requires some input data deriving from running simulations to calibrate it, but then the algorithm is able to fill the gap remain uncovered by the FE testing programme. The use of surrogate models in engineering fields is increasing and the study has demonstrated how they can be easily applied in the offshore geotechnical engineering context.

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